

Splitting of the $\pi - \rho$ spectrum in a renormalized light-cone QCD-inspired model

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We show that the splitting between the light pseudo-scalar and vector meson states is due to the strong short-range attraction in the 1S_0 sector which makes the pion and the kaon light particles. We use a light-cone QCD-inspired model of the mass squared operator with harmonic confinement and a Dirac-delta interaction. We apply a renormalization method to define the model, in which the pseudo-scalar ground state mass fixes the renormalized strength of the Dirac-delta interaction.

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I. INTRODUCTION

The effective light-cone QCD theory [1,2] is an attempt to describe the Fock-state components of the light-front meson wave-function of bound constituent quarks, constructed recursively from the lowest Fock-state component. The lowest Fock component of the hadron is an eigenfunction of an effective mass squared operator parameterized in terms of an interaction which contains a Coulomb-like potential and a Dirac-delta term. Both terms of the interaction come from an effective one-gluon-exchange and the Dirac-delta corresponds to the hyperfine interaction. This effective theory describes with reasonable success the masses of the ground state of the pseudo-scalar mesons and in particular the pion structure [3], inspite the small number of free parameters, which is only the canonical number plus one — the renormalized strength of the Dirac-delta interaction. The model does not have confinement, thus the study of excited states above the breakup threshold were not possible so far. The next step is to introduce the confining interaction, therefore one should ask as well, how the nice renormalization features of the model changes and what are the implications of this model for the spectrum of the pseudoscalar mesons.

In this work, we show an extension of the light-cone QCD-inspired theory to include the confining interaction and we perform the renormalization of the model, using as input the light pseudo-scalar masses to fix the renormalized strength of the Dirac-delta interaction, active in the 1S_0 -channel. We apply the extended model to study the splitting of the excited pseudo-scalar states from the excited 3S_0 vector meson states as a function of the ground state pseudo-scalar mass. We show that the $\pi - \rho$ mass splitting, due to the attractive Dirac-delta interaction, is the source of the splitting between the masses of the excited states, and inspite of the simplicity of the model, there is a reasonable qualitative agreement with the experimental values [4].

The non trivial mass splitting between the π and ρ meson spectrum is a consequence of the small pion mass in the hadronic scale, which fix the renormalized strength of the Dirac-delta interaction. We must point out that the renormalized strength of the Dirac-delta interaction, fitted to the pion mass, puts into the model the complex short-range physics, which makes the pion a strongly bound system. In this first work, the Coulomb-like and the confining interactions are substituted by an harmonic oscillator potential, which allowed an analytic formulation, as we are going to show. The parameters of the confining interaction in the mass squared operator, are fitted to the 3S_1 -meson ground state mass and to the slope of the trajectory of excited states with the principal quantum number [5]. We expect that the detailed form of the interaction using the Coulomb-like potential and a specific form of confinement will have a small overall effect and will not change our conclusions about the splitting of the pion and kaon spectrum in respect to the correspondent vector meson spectra.

We should add that, the observation of the almost linear relationship between the mass squared of excited states with n , as pointed out in Ref. [5], connects nicely and quite naturally with our model. The slope of the trajectory

in the (n, M^2) plane defines the harmonic oscillator strength of the mass squared operator in our model. Here, we reveal some of the physics that are brought by the work of Ref. [5] and we show the relation between the π and ρ spectrum, through the pion mass scale, which defines the renormalization condition of the model.

This paper is organized as following. In section II, we review the light-cone QCD-inspired theory for the mass squared operator for a constituent quark-antiquark system and extend it to include a confining interaction. We show how to renormalize the theory using the subtracted equations for the transition matrix of the model. In section III, we present a pedestrian approach to the theory developed in section II using an harmonic oscillator interaction in the mass squared operator equation, and show the results within this model. In section IV, we made several remarks on the qualitative aspects of the model and we give the summary of our work.

II. EXTENDED LIGHT-CONE QCD-INSPIRED THEORY

In this section we extend the renormalized effective QCD-theory given in Ref. [3] to include confinement. The bare mass operator equation for the lowest Light-Front Fock-state component of a bound system of a constituent quark and antiquark of masses m_1 and m_2 in the spin 0 channel, is described as [1,2]

$$M^2\psi(x, \vec{k}_\perp) = \left[\frac{\vec{k}_\perp^2 + m_1^2}{x} + \frac{\vec{k}_\perp^2 + m_2^2}{1-x} \right] \psi(x, \vec{k}_\perp) - \int \frac{dx' d\vec{k}'_\perp \theta(x') \theta(1-x')}{\sqrt{x(1-x)x'(1-x')}} \left(\frac{4m_1 m_2}{3\pi^2} \frac{\alpha}{Q^2} - \lambda - W_{conf}(Q^2) \right) \psi(x', \vec{k}'_\perp), \quad (1)$$

where M is the mass of the bound-state and ψ is the projection of the light-front wave-function in the quark-antiquark Fock-state. The confining interaction is included in the model by $W_{conf}(Q^2)$. The momentum transfer Q is the space-part of the four momentum transfer, the strength of the Coulomb-like potential is α and λ is the bare coupling constant of the Dirac-delta interaction. For convenience the mass operator equation is transformed to the instant form representation [6,7], with

$$x(k_z) = \frac{(E_1 + k_z)}{E_1 + E_2}, \quad (2)$$

and the Jacobian of the transformation of (x, \vec{k}_\perp) to \vec{k} is:

$$dx d\vec{k}_\perp = \frac{x(1-x)}{m_r A(k)} d\vec{k}, \quad (3)$$

with the dimensionless function

$$A(k) = \frac{1}{m_r} \frac{E_1 E_2}{E_1 + E_2}, \quad (4)$$

and the reduced mass $m_r = m_1 m_2 / (m_1 + m_2)$. The individual energies are $E_i = \sqrt{m_i^2 + k^2}$ ($i=1, 2$) and $k \equiv |\vec{k}|$.

The mass operator equation in instant form momentum variables is given by:

$$M^2 \varphi(\vec{k}) = [E_1 + E_2]^2 \varphi(\vec{k}) - \int d\vec{k}' \left(\frac{4m_s}{3\pi^2} \frac{\alpha}{Q^2 \sqrt{A(k)A(k')}} - \frac{\lambda}{m_r \sqrt{A(k)A(k')}} + \frac{W_{conf}(Q^2)}{m_r \sqrt{A(k)A(k')}} \right) \varphi(\vec{k}'), \quad (5)$$

where $m_s = m_1 + m_2$, the phase-space factor is included in the factor $1/\sqrt{A(k)A(k')}$, $\sqrt{x(1-x)}\psi(x, \vec{k}_\perp) = \sqrt{A(k)}\varphi(\vec{k})$. The momentum transfer is chosen in a rotationally invariant form as $Q^2 = |\vec{k} - \vec{k}'|^2$.

The bound state masses comes from the diagonalization of the mass squared operator, however the mass operator of Eq.(5) is ill-defined mathematically. The bound states masses are, as well, the poles of the Green's function of the theory. In view of the necessary regularization and renormalization of the model due to the singular interaction, we use the renormalization method of singular interactions developed in the context of nonrelativistic scattering equation [8] to derive the T-matrix and from which we obtain the poles of the Green's function.

A. Mass Operator and Green's Function

The operator form of Eq.(5) is written as:

$$(M_0^2 + V + V^\delta + V_{conf}) |\varphi\rangle = M^2 |\varphi\rangle, \quad (6)$$

where the free mass operator, $M_0 (= E_1 + E_2)$, is the sum of the energies of quark 1 and 2, V is the Coulomb-like potential, V^δ is the short-range singular interaction and V_{conf} gives the quark confinement. According to Eq.(5) the matrix elements of these operators are given by:

$$\langle \vec{k} | V | \vec{k}' \rangle = -\frac{4m_s}{3\pi^2} \frac{1}{\sqrt{A(k)}} \frac{\alpha}{Q^2} \frac{1}{\sqrt{A(k')}}, \quad (7)$$

$$\langle \vec{k} | V^\delta | \vec{k}' \rangle = \langle \vec{k} | \chi \rangle \frac{\lambda}{m_r} \langle \chi | \vec{k}' \rangle = \frac{1}{\sqrt{A(k)}} \frac{\lambda}{m_r} \frac{1}{\sqrt{A(k')}}. \quad (8)$$

and

$$\langle \vec{k} | V_{conf} | \vec{k}' \rangle = \frac{1}{\sqrt{A(k)}} \frac{W_{conf}(Q^2)}{m_r} \frac{1}{\sqrt{A(k')}}. \quad (9)$$

The phase-space factor $A(k)$ is defined by Eq.(4), and Q^2 is the momentum transfer squared. For convenience of the formal manipulations, the form-factor of the separable singular interaction is introduced and defined by $\langle \vec{k} | \chi \rangle = 1/\sqrt{A(k)}$.

Due to the confining interaction, the complete basis states used to describe the renormalized theory are the eigenstates of the mass squared operator without the singular interaction:

$$(M_0^2 + V + V_{conf}) |n\rangle = M_n^2 |n\rangle, \quad (10)$$

where n gives the quantum numbers of the eigenstate $|n\rangle$, and M_n^2 is eigenvalue of the mass squared operator.

Our renormalization procedure is based on Green's function method, so we introduce the free Green's function of the theory, which is given by:

$$G_0(M^2) = \sum_n \frac{|n\rangle\langle n|}{M^2 - M_n^2}. \quad (11)$$

The poles of Eq.(11) give the eigenvalues of Eq.(10).

The complete Green's function of the theory includes the Dirac-delta potential, and it is solution of the following equation:

$$G(M^2) = G_0(M^2) + G_0(M^2) V^\delta G(M^2). \quad (12)$$

The poles of Eq.(12), on the real axis, are the eigenvalues of the mass squared operator of the complete theory given by Eq.(6).

B. Renormalized T-matrix

In close analogy to scattering theory one can introduce the transition-matrix from which the full Green's function of the theory can be derived:

$$G(M^2) = G_0(M^2) + G_0(M^2) T_{\mathcal{R}}(M^2) G_0(M^2), \quad (13)$$

where the matrix elements of the renormalized transition matrix, $T_{\mathcal{R}}(M^2)$, satisfy the subtracted form of the Lippman-Schwinger equation [3,8]:

$$\begin{aligned} \langle n | T_{\mathcal{R}}(M^2) | n' \rangle &= \langle n | T_{\mathcal{R}}(\mu^2) | n' \rangle \\ &+ \sum_m \langle n | T_{\mathcal{R}}(\mu^2) | m \rangle \left(\frac{1}{M^2 - M_m^2} - \frac{1}{\mu^2 - M_m^2} \right) \langle m | T_{\mathcal{R}}(M^2) | n' \rangle. \end{aligned} \quad (14)$$

The subtraction point μ^2 is arbitrary, and it can move without affecting the results of the theory, due to that physical constraint $T_{\mathcal{R}}(\mu^2)$ changes according to the Callan-Symanzik equation

$$\frac{d}{d\mu^2} \langle n|T_{\mathcal{R}}(\mu^2)|n' \rangle = - \sum_m \langle n|T_{\mathcal{R}}(\mu^2)|m \rangle \left(\frac{1}{\mu^2 - M_m^2} \right)^2 \langle m|T_{\mathcal{R}}(\mu^2)|n' \rangle . \quad (15)$$

To solve Eq.(14), the T-matrix at the subtraction point should be found, taking into account the operator structure of the singular interaction, which is the separable form of Eq.(8), we write that

$$\langle n|T_{\mathcal{R}}(\mu^2)|n' \rangle = \langle n|\chi \rangle \lambda_{\mathcal{R}}(\mu^2) \langle \chi|n' \rangle , \quad (16)$$

where $\lambda_{\mathcal{R}}(\mu^2)$ is the renormalized strength of the Dirac-delta interaction, which is fixed by the ground state of the pseudo-scalar, 1S_0 , meson.

The solution of the subtracted scattering Eq.(14) with the T-matrix at the subtraction point defined by Eq.(16) is easily found, and it reads:

$$\langle n|T_{\mathcal{R}}(M^2)|n' \rangle = \langle n|\chi \rangle t_{\mathcal{R}}(M^2) \langle \chi|n' \rangle , \quad (17)$$

where the reduced matrix element is

$$t_{\mathcal{R}}^{-1}(M^2) = \lambda_{\mathcal{R}}^{-1}(\mu^2) + \sum_m |\langle \chi|m \rangle|^2 \left(\frac{1}{\mu^2 - M_m^2} - \frac{1}{M^2 - M_m^2} \right) . \quad (18)$$

The divergence in the sum presented in Eq.(18), is exactly cancelled by the difference between the free Green's function appearing in that equation. The reduced matrix element, $t_{\mathcal{R}}(M^2)$, is as well, the result of the change in the renormalized coupling constant due to the dislocation of the subtraction point from μ to M . The masses of the interacting system are given by the zeros of $t_{\mathcal{R}}^{-1}$ which defines the zero angular momentum states of the bound quark-antiquark systems.

The physical condition given by the ground-state of the pion or the other pseudo-scalar mesons are the input to define the reduced matrix element of Eq.(18). Choosing the subtraction point at the mass of the ground state of the pseudo scalar meson, for example at the pion mass, this implies that the renormalized strength is zero and

$$t_{\mathcal{R}}^{-1}(M^2) = \sum_m |\langle \chi|m \rangle|^2 \left(\frac{1}{\mu^2 - M_m^2} - \frac{1}{M^2 - M_m^2} \right) , \quad (19)$$

which has a zero at the mass μ . The value of μ , as the physical input, can be changed to study the splitting the 1S_0 and 3S_0 spectrum in a continuous way, as we are going to discuss in the next section.

III. SPLITTING OF THE SPECTRUM IN A PEDESTRIAN APPROACH

Now to allow an analytic treatment of the renormalized theory, while keeping its physical content, we use equal mass constituent quarks and simplify Eq.(5) to the form:

$$(M_{h.o.}^2 + g\delta(\vec{r})) \varphi(\vec{r}) = M^2 \varphi(\vec{r}) , \quad (20)$$

where the bare strength of the Dirac-delta interaction is g , and the mass squared operator for the harmonic potential is

$$M_{h.o.}^2 = 4(k^2 + m_q^2) + \frac{1}{64}w^2r^2 + a , \quad (21)$$

in units of $\hbar = c = 1$.

The eigenvalue Eq.(10) is given now by

$$\left(4(-\nabla^2 + m_q^2) + \frac{1}{64}w^2r^2 + a \right) \Psi_n(\vec{r}) = M_n^2 \Psi_n(\vec{r}) , \quad (22)$$

where n is the principal quantum number of the s-wave excited states and $\Psi_n(\vec{r})$ is corresponding to the eigenstate of the harmonic oscillator.

The observation of the almost linear relationship between the mass squared of the ρ excited states with n , as pointed out in Ref. [5], connects quite nicely to our model. The slope of the trajectory in the (n, M^2) plane defines w for each system, and the ground state mass determines a . The constituent quark mass, m_q , is included by the mass of the vector meson ground-state. In this pedestrian approach, we are using that the constituent masses of the up-down and strange quarks are degenerated and due to that we will fit each case separately. The eigenvalues of Eq.(22) are given by $M_n^2 = nw + M_{(v.g.s)}^2$, where $M_{(v.g.s)}$ is the ground-state mass of the 3S_1 meson. (In the work of Ref. [5], in the place of our n it is used $n - 1$.)

The reduced T-matrix of our pedestrian approach turns to be

$$t_{\mathcal{R}}^{-1}(M^2) = (2\pi)^3 \sum_n |\Psi_n(0)|^2 \left(\frac{1}{\mu^2 - M_n^2} - \frac{1}{M^2 - M_n^2} \right), \quad (23)$$

and the value of the s-wave eigenfunction at the origin, is given by [9]:

$$\Psi_n(0) = \alpha^{\frac{3}{2}} \left[\frac{2^{2-n} (2n+1)!!}{\sqrt{\pi} n!} \right]^{\frac{1}{2}}, \quad (24)$$

where α^{-1} is the oscillator length.

The final form the reduced T-matrix of our pedestrian model is

$$t_{\mathcal{R}}^{-1}(M^2) = (2\pi\alpha)^3 \sum_{n=0}^{\infty} \frac{2^{2-n} (2n+1)!!}{\sqrt{\pi} n!} \left(\frac{1}{\mu^2 - nw - M_{(v.g.s)}^2} - \frac{1}{M^2 - nw - M_{(v.g.s)}^2} \right). \quad (25)$$

The zeros of Eq.(25) gives the eingevales of the the mass squared operator of Eq.(20).

The parameters of the present model are w , the ground state mass of the 3S_1 meson and the renormalized strength of the Dirac-Delta interaction. We have here, the same number of parameters of the original model [1–3], which is the canonical number of 7 (α and 6 constituent quark masses) plus one, the strength of the renormalized Dirac-delta interaction. In the next, we show the numerical results for the splitting of the $\pi - \rho$ and $K - K^*$ spectrum, due to the Dirac-delta interaction acting in the pseudo-scalar 1S_0 channel.

In Figure 1, we present our results for the $\pi - \rho$ mass splitting for the first six levels as a function of the ground-state pseudo-scalar mass μ , which interpolates from the pion to the rho meson spectrum. The value of the parameter $w = 1.39 \text{ GeV}^2$ is taken from Ref. [5]. The large splitting which happens in the ground state, is diminished in the excited states, although the model attributes consistently smaller masses for the 1S_0 states compared to the respective 3S_1 ones. The $\rho(1700)$ is suggested to be a D-wave meson [5], and in the figure it does not fit well in our picture for 3S_1 meson resonance.

Although, it was pointed out [5] that the kaon trajectory is more uncertain, we have addressed here the splitting between the 1S_0 and 3S_1 excited states, because our objective is to verify the consistency of the model. The results for the kaon states 1S_0 and 3S_1 , K and K^* , are presented in figure 2. These calculations confirms the pattern which we have observed in figure 1: the large difference of the masses of the ground states translates to a much smaller difference for the excited states, and the masses of the 1S_0 states are consistently below the correspondent 3S_1 meson masses. The oscillator parameter for the kaon system is fitted to the difference of the masses squared of the $K^*(1410)$ and $K^*(892)$, with value of $w = 1.19 \text{ GeV}^2$.

IV. REMARKS ON THE QUALITATIVE ASPECTS OF THE MODEL AND SUMMARY

The pseudo-scalar mass spectrum represents a true challenge to theoretical modelling outside the physics described by just the confining interaction [10]. The splitting between the $\pi - \rho$ mass, and the huge mass difference of the π to its first resonant state $\pi(1300)$, have to be found from the fitting of the potential parameters with the hyperfine (short-range) interaction specially tuned to the pion mass. The successful description of the splitting among the S-wave hyperfine pairs $\pi - \rho$, $K - K^*$ and so on, depends crucially on the structure of the short-range part of the hyperfine potential, as concluded in the seminal work by Godfrey and Isgur [10]. The work of Anisovich, Anisovich and Sarantsev [5] showing the trajectories of the mesons in the (n, M^2) and (J, M^2) planes, putted a phenomenological order to the meson spectrum, although the driven physics of this nice observation was not clear. In regard to this last aspect, we believe that the present work reveals some of the physics of the work in Ref. [5].

In essence, the main physics of our model are given by two ingredients: the dynamics described by a mass squared operator and the Dirac-delta interaction, which brings to the model the small pion mass. We should point out that in

the literature, the attractive short-range interaction has been extensively addressed in the study of the meson structure and spectrum. Below, we will discuss very briefly some of these previous works, that are more closely related to the present one in that particular aspect.

The Godfrey and Isgur model was used to construct light-cone wave-functions for the pion, rho-meson and nucleon [11], and it was pointed out that a high-momentum tail in the wave-function above 1 GeV/c [11] is due to the short-range attraction. The impact on the structure of the pion and rho mesons were tested in electroweak observables with some success [11]. However, it was pointed out [12] that the existing electroweak data was not sensitive enough to allow a definite phenomenological conclusion about the presence of hard-constituent quarks in the hadron wave-function.

Another approach to the meson spectrum within a covariant quark model [13], using the framework of instantaneous Bethe-Salpeter equation, also pointed out the importance of the short-range attraction in the meson spectrum, due to the large mass splittings in the pseudo scalar ($J = 0$) sector, which in that work was attributed to the two-body point-like part of 't Hooft's instant induced $U_A(1)$ symmetry breaking interaction [14].

The structure of the pion light-cone wave function, has been recently probed experimentally by diffractive dissociation of 500 GeV/c π^- into dijets which provided a direct measurement of the pion valence-quark momentum distribution for the first time [15]. The result give a stringent test of the asymptotic wave-function [16], which matches the high momentum tail of the pion wave-function [3]. In that sense, this experiment supports our model with the singular interaction it contains.

From the previous discussion, where we just mentioned some examples, becomes clear the necessity of a strong short-range attractive interaction to built the pion, as seen in the modelling of the light pseudo scalar meson spectrum and in the recent measurement of the pion valence wave function. In that sense, we believe that the Dirac-delta interaction parameterizes the complex short-range physics of Quantum-Chromodynamics, which is brought to the model by the small pion mass. The splitting of the pion and kaon spectrum from the respective 3S_1 -meson counterparts, are a direct consequence of the Dirac-delta interaction and its renormalization, the structure of the mass squared operator equation with confinement, with all the relativistic effects implied by it.

In summary, we extended the renormalized light-cone QCD-inspired effective theory to include confinement in the mass squared operator, which has a Dirac-delta interaction. We applied the model to study the splitting of the $\pi - \rho$ and $K - K^*$ spectrum, using a pedestrian approach to it, with an harmonic oscillator interaction. In this way, we derived an algebraic equation, where the solutions give the mass squared of the 1S_0 states, as a function of the ground-state of the pseudo-scalar meson. The model accommodates naturally the large splitting of the ground state of the light 1S_0 and 3S_1 and its suppression in the excited states, in qualitative agreement with the experimental data. At the same time the model provides a sound physical basis to understand the systematics of the $q\bar{q}$ -states in the (n, M^2) plane and as well in the (J, M^2) plane. Therefore, we have shown that the extension of the light-cone QCD-inspired model to include confinement while keeping its simplicity and renormalizability, provides a reasonable picture of the spectrum of light mesons.

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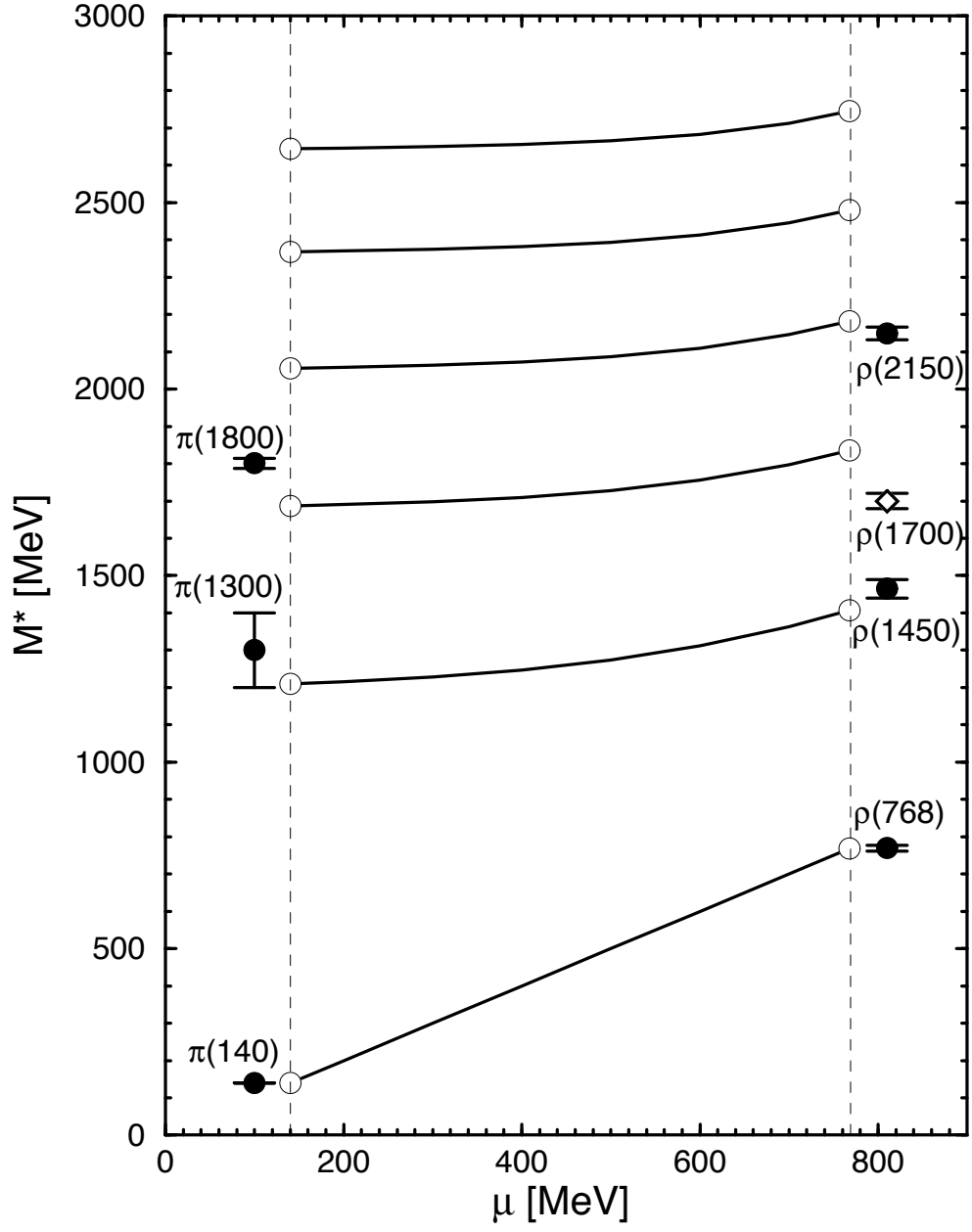


FIG. 1. Mass of the excited $q\bar{q}$ state (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I = 1$. Numerical results for M^* from the zeros of Eq.(25). The experimental data comes from Ref. [4]. The data on the left and right of the figure corresponds to the 1S_0 and 3S_1 mesons, respectively.

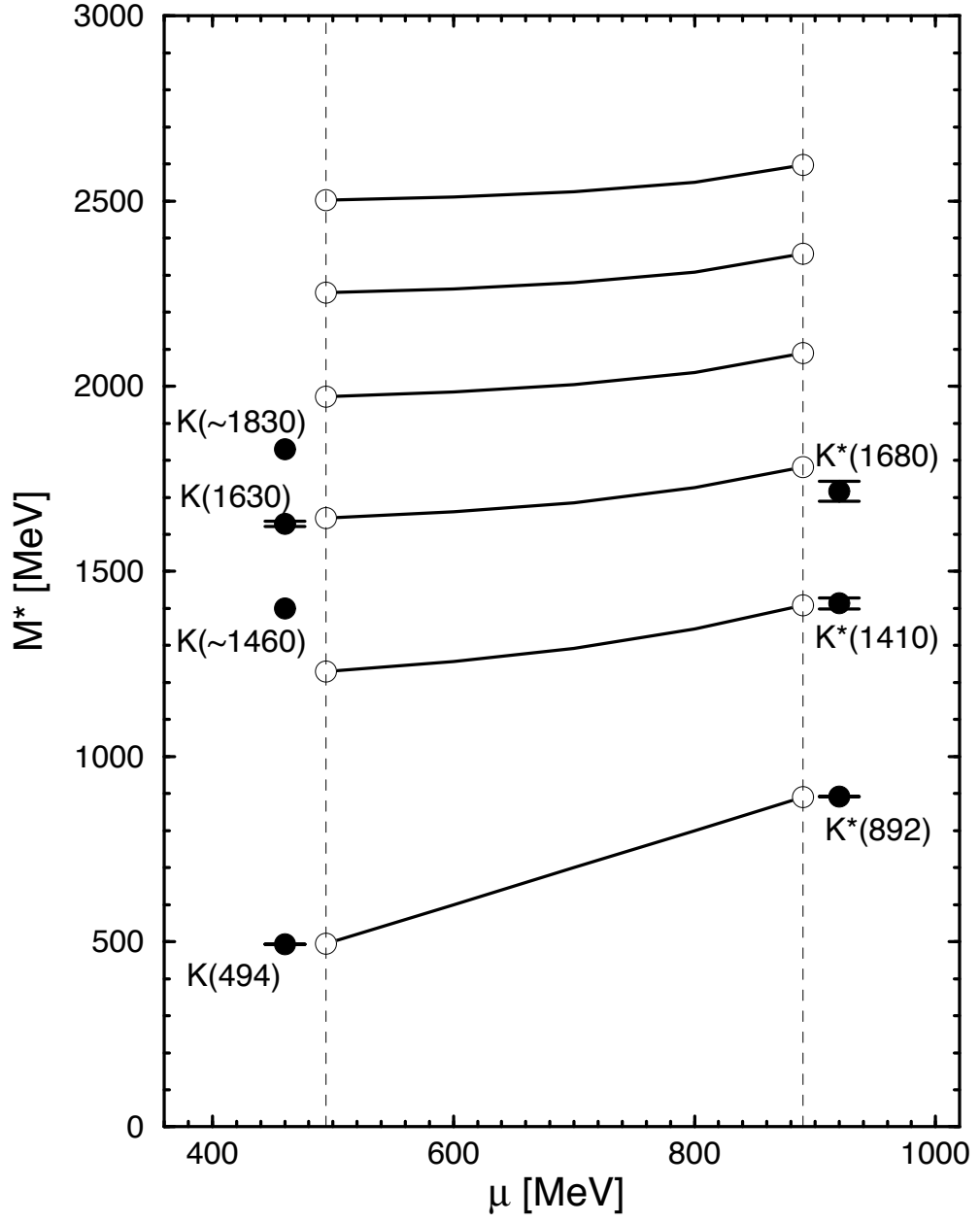


FIG. 2. Mass of the excited $q\bar{q}$ state (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I = 1/2$ of the strange sector. Numerical results for M^* from the zeros of Eq.(25). The experimental data comes from Ref. [4]. The data on the left and right of the figure corresponds to the 1S_0 and 3S_1 mesons, respectively.